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Corner smoothing for CNC machining of linear tool path: A review

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Abstract: Computer numerical control (CNC) machine tools are widely used in various industrial fields ranging from aerospace, automotive, ship building, and the die/mould to manufacture products. However, tool paths of most CNC machine tools are composed of a series of linear motion commands (G01), which will inevitably cause the discontinuity in curvature and feedrate at the junctions between adjacent linear tool path segments, deteriorating the surface quality with unfavorable marks and decreasing the machining efficiency. To solve this problem for obtaining the steady and continuous motions of machine tools, the local corners have to be smoothed. Generally, the existing corner smoothing methods can be classified as the global smoothing and the local corner smoothing, where the specially designed transition curves or the directly planned motion of machine tools are adopted to generate the geometrically corner-free tool path. Moreover, some new methods that focus on developing different transition or rounding strategies are also developed for further improving the kinematics performance of machine tools. In this paper, the recent advances and researches on corner smoothing methods are reviewed from different categories, and the conclusions, remaining challenges and future directions in corner smoothing are also presented.

Keywords: CNC machining; Robot machining; Linear tool path; Global smoothing; Local smoothing; Corner rounding

1. Introduction

Computer numerical control (CNC) machine tools are widely used in industrial manufacturing to achieve high-efficiency, high-precision and high-performance machining of various parts [1-3]. In the typical machining process of these high standards and strict requirements parts, a huge number of linear motion commands (G01), which are the most commonly used format to approximate the tool paths in the existing CNC machining, are designed by the computer-aided manufacturing (CAM) software to accomplish the manufacturing processes [4-6]. However, such discrete linear tool path will inevitably cause the discontinuity in curvature and feedrate at their junctions, resulting in the deterioration of surface quality and the decrease of machining efficiency [7-9]. Thus, smoothing the linear tool path has captured extensive attentions from researchers.

Recently, a great deal of corner smoothing methods haven been proposed to round the corners for generating the geometrically corner-free tool path, where the global smoothing methods [10-12] and the local corner smoothing methods [13-15] are the most widely used according to their smoothing scopes/extents. The global smoothing fits all discrete tool path positions, which can guarantee the global smoothness of the linear tool path. Contrarily, the local corner smoothing blends the adjacent linear tool path segments by replacing the local corners, which can not only achieve the high-order continuity at the junctions between the inserted transition curves and linear segments, but also accurately control the approximation error specified by the users at the corners. Thus, based on the properties of these two methods, the spline based geometry methods (also known as two-step methods) are studied more frequently in local smoothing methods, such as Bézier spline [16-19], B-spline [20-23], non-uniform rational B-spline (NURBS) [24-26], Pythagorean Hodograph (PH) curve [27-29], Akima curve [30] and clothoid spline [31-33], etc. While, with the rapid development of sensors and filtering technologies, direct motion planning based [34-36] and finite impulse response (FIR) filter based [37-41] kinematics methods (also known as one-step methods) are most involved in the global smoothing. The spline based geometry methods are first to specially insert transition curves to replace the corners, then schedule the feedrate of machine tools. However, the kinematics methods are to directly plan the motions of machine tools without constructing the transition curve in advance to eliminate the corners. Thanks to the simplicity and convenience in constructing the splines, the research works related to the local corner smoothing are little more extensive than that of the global smoothing. But to some extent, benefit from the smoothing of entire tool path, the global smoothing usually can obtain better kinematics performance of machine tools than local corner smoothing.

Nowadays, corner smoothing has gradually transformed from a single corner-free level to the performance-oriented level. For ex-

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ample, asymmetrical transition curves [42-46] and asymmetrical motion profiles [47, 48] have been fully concerned, since they can independent adjust the transition profiles on both sides of the corner when compared with the traditional symmetrical ones. So that the newly constructed transition profiles using these asymmetrical transition methods will not be excessively constrained to over-approach the corners, leading to the decrease of cornering curvatures and the improvement of the feedrate at the corners. Similarly, the circumscribed corner smoothing methods [49-51] have been proposed to against with the most widely used inscribed methods, since the circumscribed transition curves can also decrease the curvature of transition curves under the user-specified approximation error and improve the feedrate at the corners. Consequently, both the asymmetrical transition and circumscribed rounding methods can be adopted to further improve the kinematics performance of machine tools through adjusting the symmetry or rounding form of transition profiles. In the meantime, real-time corner smoothing methods are also being carefully studied [52, 53] due to their practicality. At present, they are mainly constituted by three aspects of algorithms, i.e., look-ahead interpolation based algorithms [54-58], analytical representation based algorithms, such as PH spline [59-61], and online FIR filter based algorithm [62-66]. Look-ahead interpolation based algorithms can reduce the computational burden by preprocessing the following linear commands, making up for the ability of CNC system in handling numerous of tool path at the same time. The analytical representation can alleviate the computation time of algorithm by avoiding or reducing the adaption of iterative processes, thus, satisfying the requirement of machine tool interpolation cycle. While the FIR filters can manipulate the cornering trajectory of the machine tool by filtering the discontinuous axis velocity commands at the junctions between adjacent linear segments. The details will be introduced in the following sections. As a result, the real-time corner smoothing methods are more conducive to improving the machining efficiency of linear tool path. Lately, a support vector machine algorithm based on machine learning was proposed by Zhou et al. [67] to classify the tool path continuity in corner smoothing, which brought a new approach by integrating the intelligent algorithms into traditional methods. However, the factors or conditions for corner smoothing of different actuators can hardly be the same, such as three-axis machining, five-axis machining and robot machining have their own concerns, which also should be taken into account in developing the corresponding corner smoothing methods.

At the early stage, corner smoothing was developed in three-axis machining to improve the machining efficiency and quality of parts [68-71], which only the tool path of tool tip position should be considered. Afterwards, dedicated to adapting to the various types of tool path, corner smoothing is gradually expanded to four-axis machining [72, 73] and five-axis machining [74-77]. Due to the additional adoption of rotary axes, the smoothness of the tool orientation should also be paid enough attention. Recently, the corner smoothing of flying robots [78], five-DOF (degree of freedom) [79, 80] and six-DOF robot machining [81-85] has been developed after ongoing researches. With the increase of DOF, the tool orientation or the joints also need to be smoothed simultaneously in addition to the tool tip position. Summarily, according to the above descriptions, the existing corner smoothing methods can be classified into multi categories from different perspectives, as shown in Fig. 1. To clearly introduce the existing corner smoothing methods, this review mainly focuses on more recent studies in the last decade from these categories and is organized as follows. In Section 2, global and local corner smoothing are discussed in detail from the perspective of geometry and kinematics methods. Section 3 mainly focuses on those newly developed corner smoothing methods oriented by the performances, followed by how these methods and cases are implemented on the machine tools and robots in Section 4. Finally, the conclusions are presented in Section 5.

2. Global smoothing and local corner smoothing methods

Global smoothing and local corner smoothing are always the most widely studied methods. They are the earliest defined and it can be said that the existing corner smoothing methods are all a branch of them. With the evolution of corner smoothing methods in recent years, geometry methods based on spline construction and kinematics methods based on motion profile planning are becoming more and more prevalent among global smoothing and local corner smoothing. This section mainly focuses on how to use these methods to achieve a geometrically corner-free tool path.

2.1. Global smoothing based on geometry methods

The global smoothing methods usually fit all discrete tool path positions by using the transition curves or transition profiles, which can guarantee the global smoothness of the linear tool path, leading

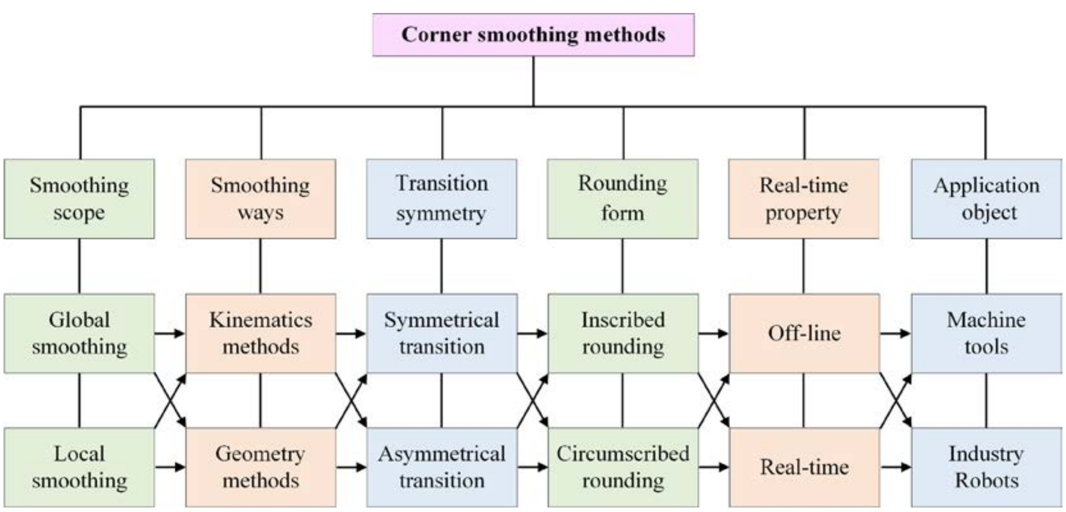


Fig. 1. Classification of corner smoothing methods.

to the steady and continuous motions of machine tools. At the beginning of global smoothing, the transition curves are first adopted to construct an entire continuous tool path. As pointed out by Erkorkmaz and Altintas [86] that the newly constructed entire splines must not only guarantee the desired tool path accurately, but also have smooth kinematic profiles in order to obtain the high tracking accuracy and avoid exciting the natural modes of the mechanical structure or servo control system. In fact, the entire spline segment provides a more continuous feed motion compared to the linear and circular segments, which can result in shorter machining time and better surface topography. For these reasons, a quintic spline trajectory that produces continuous position, velocity, and acceleration profiles were presented in Ref. 83 to smooth the linear tool path by specially designing the interpolation algorithm. Furthermore, Lei et al. [7] proposed a NURBS tool path interpolation method, where the common problem that the calculation of the total arc length of the NURBS tool path was solved by using the inverse length functions for each resulting subinterval, thus the time-consuming computation of NURBS derivatives were avoided and a global smoothing tool path can be generated. Meanwhile, Yau and Wang [87,88] also adopted the entire splines to smooth the linear tool path, in which the fast interpolation methods were proposed to fit the continuous linear tool path into transition curves, and then by scheduling the complete feedrate with the S-shaped jerk-limited acceleration profile (JLAP), the smoother tool path can be directly constructed at the same time.

In addition to the feedrate scheduling and interpolation algorithms in constructing the transition curves to guarantee the global smoothness of the linear tool path, the manipulation of control points of entire spline is also studied to achieve this goal. As shown in Fig. 2, Du et al. [11] employed a B-spline curve to approximate the tool path within the desired fitting accuracy by appropriately determining the distribution of the dominant points, after mathematically calculating the control points of transition curve, the global smoothing tool path can be effectively generated. To simplify the calculation of control points, a global corner smoothing method was proposed by Wang et al. [12], in which the control points of transition curve were solved without the matrix operations and the number of control points was also reduced while satisfying the interpolation error. However, as reported by Du et al. [11], the fitting error of global tool path smoothing is not easy to predict and control accurately, the new corner smoothing methods that can constrain the fitting error under a desired accuracy need to be carefully studied.

2.2. Global smoothing based on kinematics methods

For further controlling the fitting error without preliminary constructing the transition curves, the direct motion planning based and FIR filter based kinematics methods are gradually employed in the global smoothing, because these methods can also guarantee the global smoothness of linear tool path while constraining the geometry and kinematics limits under a specified tolerance.

In terms of the direct motion planning based kinematics methods, the smoothed tool path can be generated by directly scheduling the feed of each drive axis on the basis of their kinematics limits. Based on this, Tajima and Sencer [34,35] proposed the kinematic corner smoothing methods for high-speed CNC machine tools to generate the continuous feed motion along the sharp corner tool path, the planned cornering profile is shown in Fig. 3. From the figure, it is not difficult to find that, instead of locally modifying the cornering tool path with a spline, the motion planning based kinematics methods can blend the axis velocity between consecutive linear tool path segments according to the user-specified cornering error and kinematics limits. Besides, the kinematics method also can be used to generate the transition trajectory through separately planning the transition trajectory and feedrate [36]. A modified corner transition strategy with more free transition space is favorable for further enhancing the machining efficiency under the axial acceleration/deceleration limits while guaranteeing the with kinematically smoother tool path.

While for the FIR filter based kinematics methods, the FIR filters are utilized to obtain the impulse response at the cornering duration to identify the corner tool path and then modulate the corresponding trajectories, as shown in Fig. 4, in which the tool tip position and tool orientation can be globally smoothed using FIR filters through blending the kinematics profiles. In order to smooth the discrete linear tool path with long segments, Liu et al. [38] proposed a kinematic method to continuously interpolate G01 commands in five-axis machine tools, in which the FIR filters were adopted to generate the time-optimal and jerk-limited trajectory of linear tool path segments by overlapping directly the adjacent kinematics profiles in one step. The similar work was also introduced in Ref. 39 to improve the machining efficiency and the smoothness of linear tool

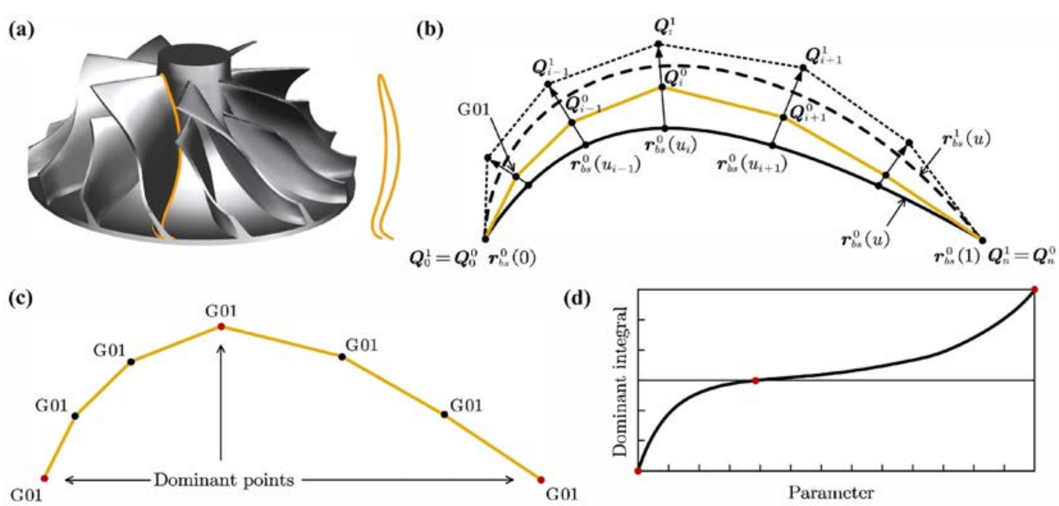


Fig. 2. Global smoothing based on geometry methods [11]. (a) Tool path of pressing wheel; (b) Progressive and iterative approximation method; (c) Dominant point method; (d) Dominant integral method.

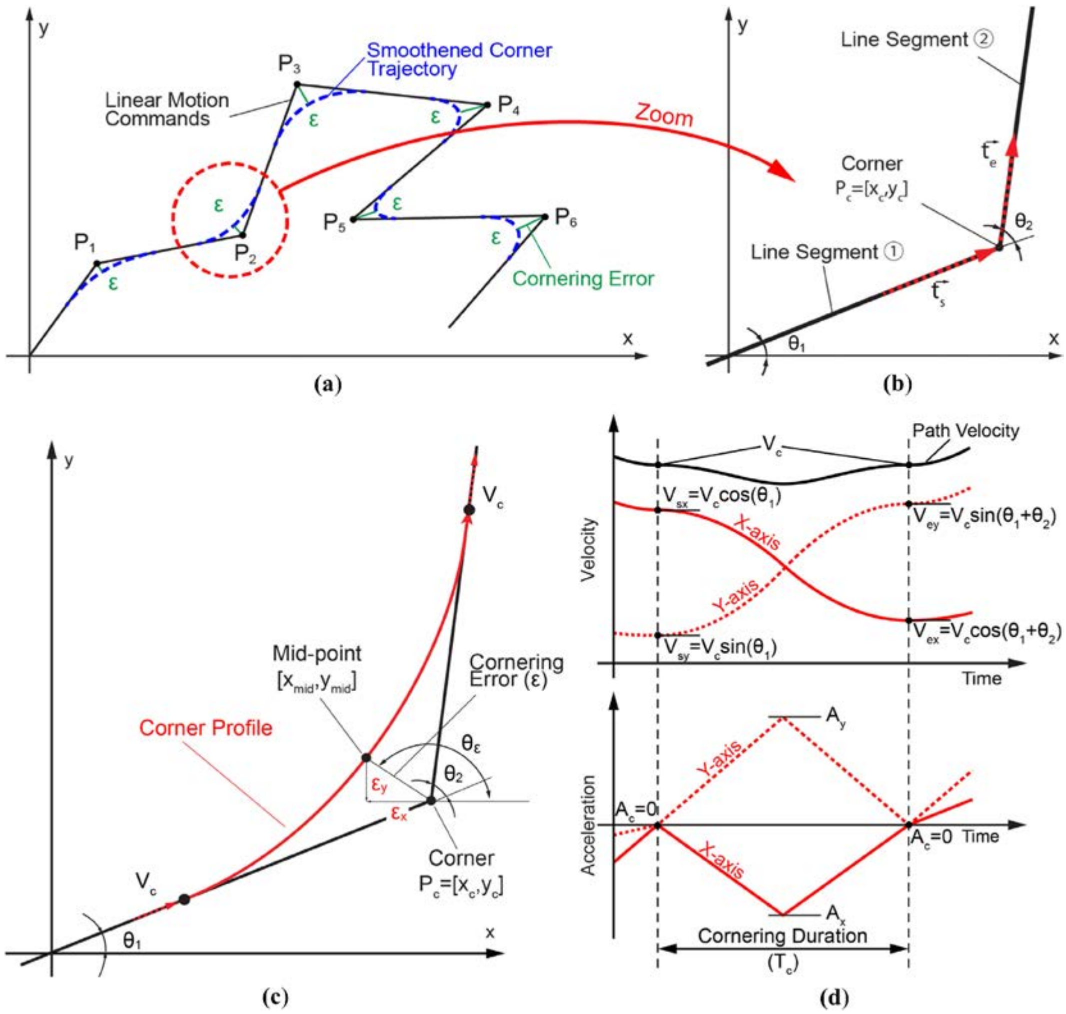


Fig. 3. Global smoothing based on the direct motion planning [34]. (a) Linear segmented tool path; (b) Sharp corner profile; (c) Smoothed corner trajectory; (d) Velocity and acceleration profiles.

path, and the only difference is that the control-point-assigning-based global geometry smoothing algorithm and a FIR filtering-based global motion smoothing algorithm were mixed together. However, unlike the corner smoothing of long-segmented tool path that only the adjacent tool path segments affect the cornering error of a specific corner, more than two tool path segments affect the error in the case of short-segmented tool path. To solve this problem, Hayasaka et al. [40] proposed a kinematics smoothing method based on FIR filter, where the short-segmented tool path can be smoothed with lower vibration, high accuracy and shorter machining time by optimizing the feedrate profiles along a span of short G01 and G02/G03 commands. Recently, a new FIR filter based corner smoothing with double filter technique was proposed to generate the interpolated trajectories for high-speed machine tools [41]. Through these methods [38-41], the appropriate filter order and corner speed under the specified kinematics constraints and contour tolerance are proved essential for corner smoothing using FIR filter based methods. Obviously, the FIR filter based kinematics methods can effectively smooth the global tool path by handling the impulse response of each drive axis while satisfying the geometry and kinematics errors.

2.3. Local corner smoothing based on geometry methods

Different from the global smoothing, the local corner smoothing uses a micro transition to blend the adjacent linear tool path segments for eliminating the local corners, which can not only achieve the high-order continuity at the junctions between transition segments and linear tool path, but also accurately control the approximation error specified by the users at the corners. Since the tool paths generated by most existing CAM applications are always consisted by a series of linear segments, and due to the simplicity and convenience of designing the transition curves between these linear tool path segments, which often requires no iterative computations to satisfy the accuracy requirement, the research works related to the local corner smoothing based on geometry methods are becoming more and more popular.

For the local corner smoothing based on geometry methods, the corner smoothing is generally completed within two steps, namely, design the parametric transition curves and plan the motions of drive axes. In the first step, a certain type of spline curves will be determined to replace the local corners of linear tool path. Theoretically, the representation of transition curve is the key point, in which the control points are often calculated according to the user-specified approximation error, overlap constraints and continuity requirement at the junctions between remaining linear tool path segment and newly inserted transition curves. In the second step, the feedrate and interpolation will be scheduled using different algorithms that are constrained by geometry and kinematics limits [75]. After finishing these two steps, the discrete linear tool path can be

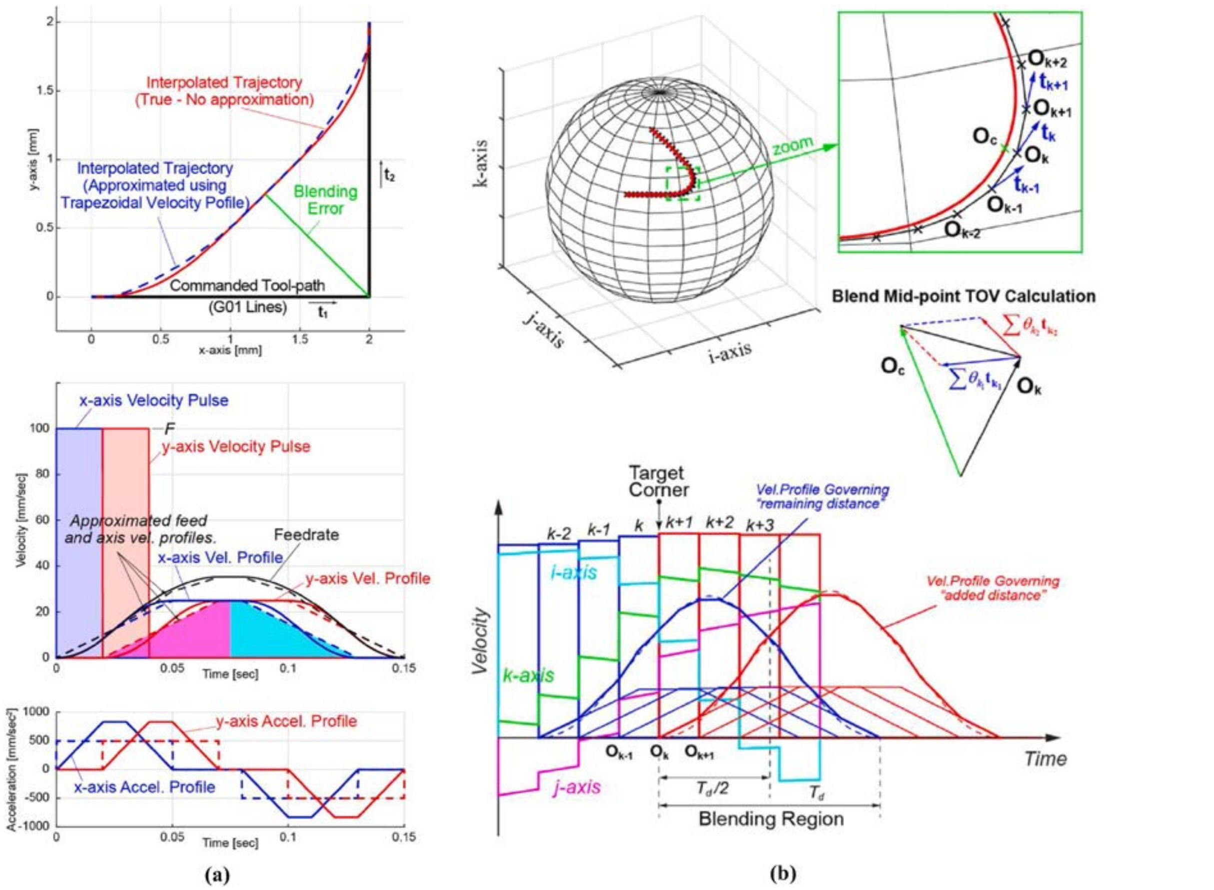


Fig. 4. Global smoothing based on the FIR filter based kinematics methods [66]. (a) Corner smoothing of the tool top position; (b) Corner smoothing of the tool orientation.

smoothed by numerous micro transition curves with the tangential continuity of curvature and feedrate guaranteed. For example, Tulsyan and Altintas [13] proposed to insert quintic and septic micro-splines at the adjacent linear tool path segments for the tool tip position and tool orientation, respectively. Optimal control points were calculated for position and orientation splines to achieve C^3 continuity at the junctions while respecting user-defined approximation error. After constructing the parametric transition curves, the smoothed corners were traveled at a smoothly varying feedrate with cubic acceleration trajectory profile. For further obtaining the higher tracking accuracy and lower acceleration frequency, Yang and Yuen [14] developed an analytical C^3 continuous corner smoothing method of five-axis tool path by locally inserting transition curves into the corners of linear tool path segments. Both the tool tip position and tool orientation were smoothed by using quintic B-spline curves, as illustrated in Fig. 5. All control points of the locally inserted tool tip position and tool orientation splines can be analytically derived according to the above-mentioned conditions. Then, by adopting the C^3 continuous cubic acceleration profile, the feedrate was scheduled and proved to be continuous. Moreover, some new aspects of local corner smoothing methods are being studied with the improvement of performance requirements in modern machining, which can maintain better the kinematics performance of machine tools while guaranteeing the machining quality. These new methods will be introduced in detail in Section 3.

2.4. Local corner smoothing based on kinematics methods

Although the most common methods of reducing the trade-off between feedrate and accuracy are to smooth the corner using a pre-specified spline that allows high-speed cornering subject to geometry error and machine kinematic limits [25], the local corner smoothing based on the direct motion planning methods still have the potential ability to obtain a kinematically smoother tool path within the given errors. Usually, kinematics methods used in local corner smoothing mainly focus on planning the motions of drive axes at the local corners, and then blend with the kinematics profiles of the remaining linear tool path segments. In fact, when using the kinematics methods to smooth the local corners, not only the maximum cornering error at the local corner under the command and servo limits has to be taken into consideration, but also the connecting feedrate between two adjacent linear tool path segments need to be scheduled. So that the overall contour errors can be bounded within the given tolerance [89]. On the basis of these, Lin et al. [90] proposed a local corner smoothing method with consideration of kinematics constraints to generate smooth motion trajectory of five-axis linear tool path. Instead of inserting a spline curve at adjacent linear segments, their method directly blended the velocities and accelerations of five motion axes and generated C^2 cornering trajectories while respecting cornering approximation error and kinematics limits of the drive axes. Moreover, corner feedrate was moderately adjusted and rotary axes feeds were synchronized with corner velocity profile to achieve the local corner smoothing. In the meantime, Nshama et al. [91] also proposed to systematically generate corner smoothed trajectories for feed drive systems through balancing the best trade-off between cycle time and energy consumption. The linear tool path geometry was subdivided into linear and corner segments, where the motions of axes on linear and corner segments were scheduled by JLAP and kinematic corner smoothing technique with interrupted acceleration, respectively. Apparently, the local corner smoothing based on kinematics methods allow as less reduction of the feedrate as possible since the motions of each axes can be smoothed separately, thereby being able to provide a relatively higher feedrate than those methods that only consider the geometry accuracy.

3. Performances oriented corner smoothing methods

Initially, the purpose of corner smoothing is to eliminate the full stop of machine tools at the local corners, so as to obtain the continuous and stable motions of machine tools. Recently, with the in-depth study of corner smoothing, more and more researchers began to explore the kinematics performance oriented methods under the premise of corner-free. This section will introduce these newly developed corner smoothing methods.

3.1. Corner smoothing using asymmetrical transition

Asymmetrical transition means the transition curve or profile is asymmetrical about the corner bisector, which can be utilized to further improve the curvature of the transition curves or radius of transition profile. Typically, the traditional local corner smoothing methods adopt the symmetrical transition curves or profiles to round the corners attribute to their convenient design (see Fig. 5). However, the adjacent transitions are likely overlapping with each other when they are initially constructed under the user-specified approximation error, which can be observed from Fig. 6(a). In this case, the transition lengths on the overlapping side of the corner have to be adjusted to eliminate the overlap, but the transition lengths on the other side of the corner will be simultaneously adjusted at the same extent due to the symmetry even if there is no overlap. Such behavior will force the transition curves to over-approach the local corners, leading to the relatively large curvatures of the transition curves and fail to increase fully the feedrate at the corners. To achieve the independent adjustment of transition lengths on both sides of the corner, Shi et al. [28] adopted an asymmetrical PH curve to smooth the linear tool path for improving the kinematics performance of machine tools, however, the potential overlaps between adjacent transition curves were not fully considered, especially for the short-segmented tool path. For further eliminating the overlap in short-segmented tool path, Bi et al. [42] proposed an asymmetrical transition algorithm for the high-speed machining of the linear five-axis tool path, where the transition lengths were constrained not exceed the half of the linear segments to avoid the potential overlaps. Actually, it has been noticed that the adjacent transition curves are prone to overlap when rounding the corners of short-segmented tool path, using such fixed-length constraint to eliminate the overlaps will excessively constrain the transition lengths, since the overlong transition lengths without overlaps will also be shortened to the half of the linear segments, which deteriorates the feedrate at the corners. More importantly, the types of overlap vary with tool path geometry, such as included angles

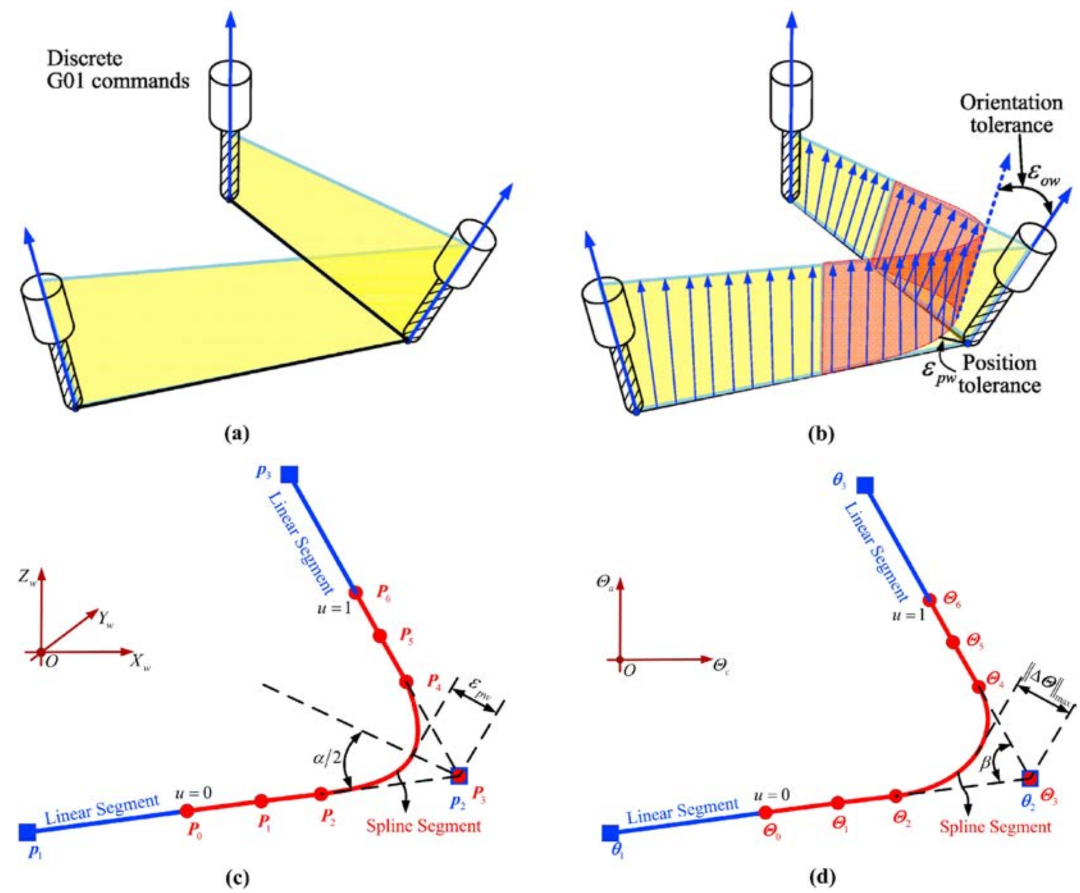


Fig. 5. Local corner smoothing with transition spline curves [14]. (a) Discrete five-axis G01 commands without corner smoothing; (b) Five-axis G01 commands after local corner smoothing; (c) Local corner smoothing of tool tip position in the workpiece coordinate system; (d) Local corner smoothing of the tool orientation in the machine coordinate system.

and lengths of linear segments. On the basis of these, an asymmetrical transition based corner smoothing method driven by overlap elimination was proposed by Yan et al. [45], where the asymmetrical transition curves were generated by adjusting independently the transition length on the overlapping side of the corner, as shown in Fig. 6(b). Besides, adopting the targeted overlap elimination strategies for different types of overlap was also turned out to be more effective to fully increase the feedrate at the corners than simply constraining the transition lengths in most existing methods, therefore, geometrically corner-free and kinematically smoother tool path with higher feedrate could be effectively generated when eliminating the overlaps between adjacent transition curves. By extending the overlap classification into corner smoothing of five-axis short-segmented tool path, Yan et al. [46] further proposed a kinematically coordinated corner smoothing method using the double asymmetrical transition curves, which can not only increase the feedrate of the machine tool at the corners, but also achieve the low acceleration and jerk of rotary axes. Recently, asymmetrical PH splines were also constructed for pursuing the analytical representation and the real-time property [43, 44], which can further reduce the running time of the local corner smoothing algorithms.

Different from the two-step corner smoothing methods that geometrically replace the sharp corner with a pre-specified asymmetrical curve. Some researchers attempted to use a kinematically asymmetrical transition profile to smooth the corners of linear tool path, the cornering trajectory can be seen in Fig. 7. For the works in Fig. 7(a), a corner smoothing method with asymmetrical transition profiles was proposed through directly scheduling the motions of each drive axis in one step, which brought good performance in cycle time and contour accuracy with the geometry error tolerance and kinematics limits guaranteed at the same time [47]. However, the linear tool path segments were assumed long enough without any overlaps in Ref. 47, which cannot be directly adopted in the short-segmented tool path. For the purpose of solving this problem, Wang et al. [48] further proposed a local asymmetrical corner trajectory smoothing method by adopting the bidirectional planning and adjusting algorithm for the situation where the smoothed cornering paths are close to each other or even overlap, which can also realize accelerated/decelerated cornering transition in one step while respecting the user-specified tolerance, as the works in Fig. 7(b). Compared with the symmetrical transition profiles with one-step method, the asymmetrical one can fully exploit the performance of the drives to improve better the kinematics, since the drive capability of each axis is different according to the inherent attribute of the machine tool.

3.2. Corner smoothing using circumscribed rounding

The inscribed and circumscribed corner smoothing methods refer to two transition forms whether the corners are rounded from the inside or outside of the linear tool path. The inscribed rounding is the widely adopted form since the transition curves can be easily constructed according to the approximation error and geometry continuity, such as the transition curve in Fig. 4, while the circumscribed rounding needs to insert extra control points or transition curves to obtain the lower curvature at the corners than that of the inscribed rounding, as shown in Fig. 8.

In circumscribed rounding works, Xu and Sun [49] proposed to adopt the double cubic B-splines to smooth the five-axis linear tool path, extra control points were used to generate the G^2 continuous circumscribed transition curves, as shown in Fig. 8(a). Compared with the commonly used inscribed corner rounding methods, the transition curves generated by the circumscribed method were confirmed with a smaller curvature, leading to a higher feedrate at the corners. In the similar way, Sun and Altintas [50] developed a corner smoothing method through using the circumscribed Bézier splines within the specified tolerance. The constructed transition curves were also proved to obtain a higher feedrate at the local corners.

Further, Yin et al. [51] pointed out that the circumscribed rounding should also taken into consideration of featured information of parts, focused on this topic, they developed a specially circumscribed corner smoothing with G01 shape-preserving for five-axis linear tool path by additionally inserting the specially designed transition curves into the local corners, as shown in Fig. 8(b). Through analyzing the conditions of G01 shape-preserving, the configuration expression of control points of transition curves was derived with original tool path position passed through. After that, the tool tip position and tool orientation were both smoothed by the circumscribed transition curves in different coordinate systems. It is noticed from above methods that the circumscribed rounding methods not only can obtain the geometrically smooth tool path under the specified approximation error, more importantly, but also can improve the curvature of transition curves together with higher fee-

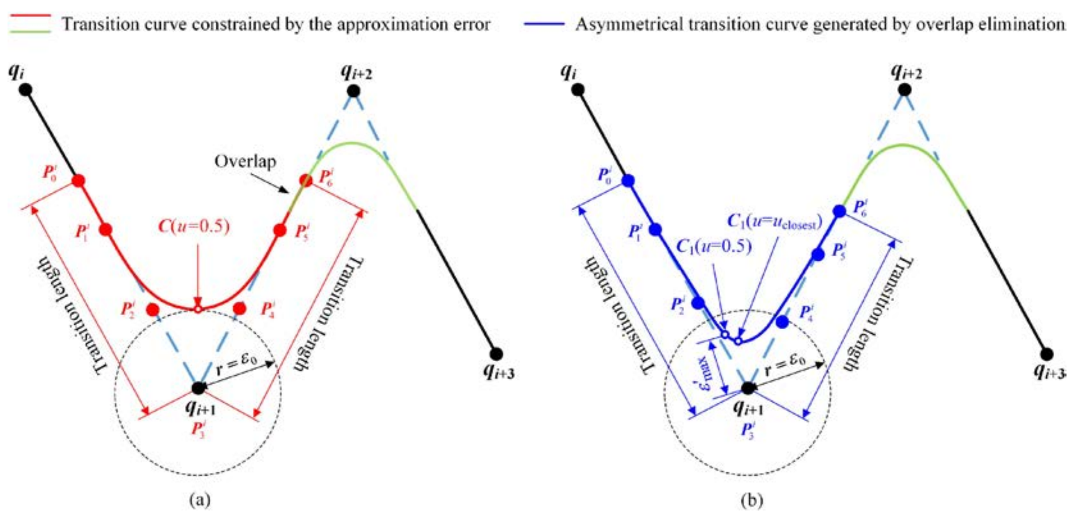


Fig. 6. Corner smoothing using asymmetrical transition curves [45]. (a) Transition curves with overlap constrained only by the approximation error; (b) Asymmetrical transition curves generated by overlap elimination.

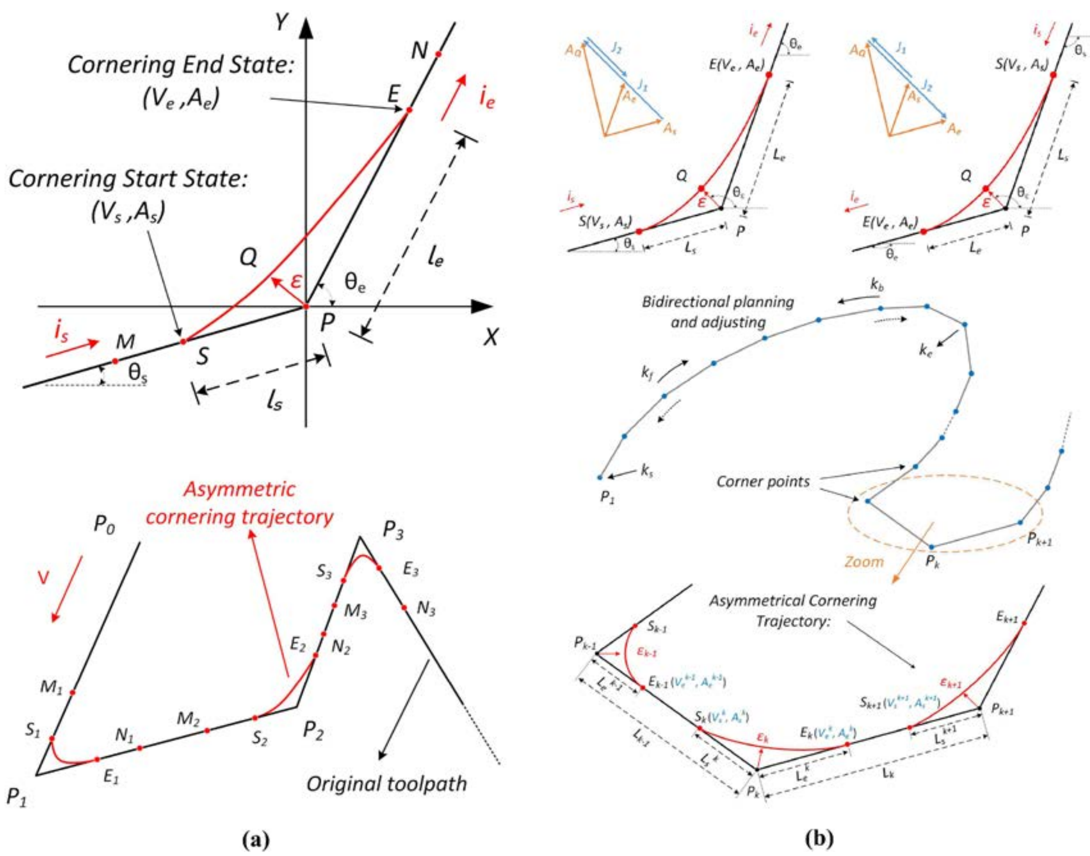


Fig. 7. Corner smoothing using asymmetrical motion profiles. (a) Asymmetrical cornering trajectory of linear tool path using direct motion planning [47]; (b) Asymmetrical corner trajectory smoothing using bidirectional planning and adjusting algorithm [48].

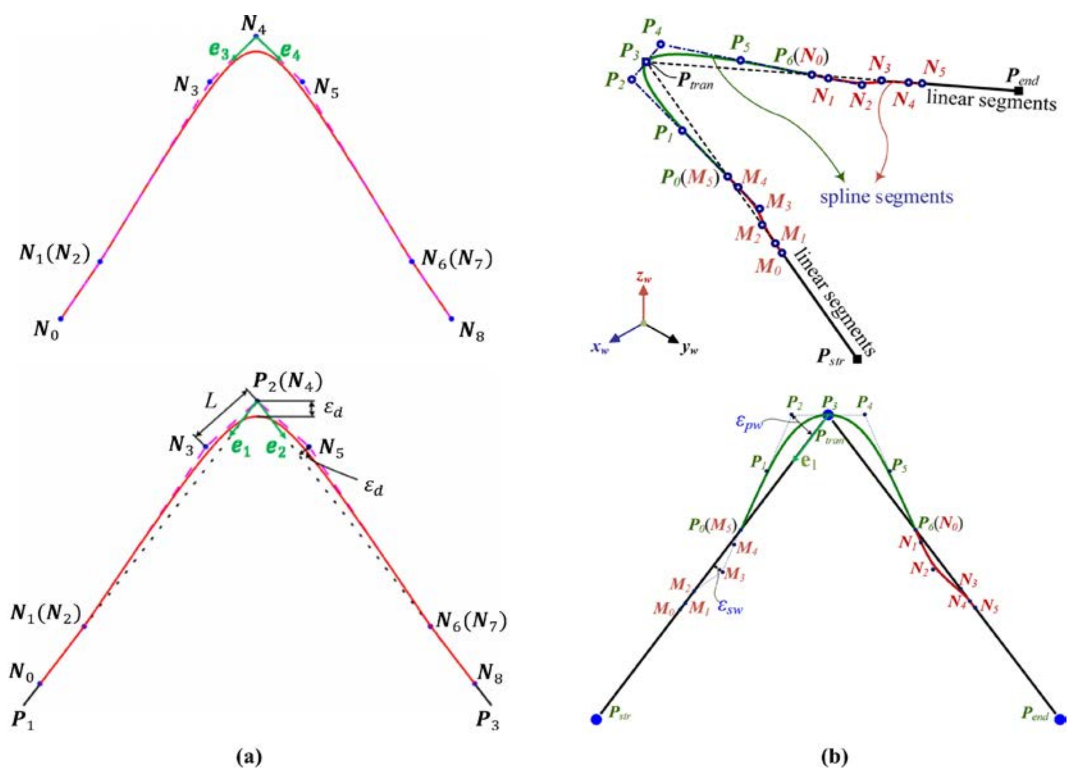


Fig. 8. Corner smoothing using circumscribed rounding. (a) Circumscribed rounding by inserting extra control points, where the generated transition curve does not pass through the tool tip position [49]; (b) Circumscribed rounding by designing extra transition curves, where the generated transition curve passes through the tool tip position [51].

drate at corners.

3.3. Real-time corner smoothing

In order to enhance the convenience and practicality of the corner smoothing methods, real-time algorithms have attracted extensive attentions from researchers, which can greatly improve the machining efficiency of parts. According to the concerned implementation related ways, the existing real-time corner smoothing methods are roughly classified into the following three categories:

(1)Real-time corner smoothing based on look-ahead interpolation. The look-ahead interpolation can reduce the computational burden through preprocessing the subsequent tool path segments, making up for the ability of CNC system in handling numerous of tool path at the same time even though the algorithm is non-analytical.

(2)Real-time corner smoothing based on analytical representation. The analytical representation can ameliorate the computation time of algorithm by avoiding or reducing the adaption of iterative processes. In this way, the processing time of algorithm can satisfy the requirement of machine tool interpolation cycle.

(3)Real-time corner smoothing based on FIR filter. The digital filtering (convolution) enables real-time interpolation of linear tool path through FIR filtering of discontinuous axis velocity commands at the junctions between adjacent linear segments, thereby the cornering trajectory of the machine tool can be directly manipulated.

In terms of the real-time corner smoothing based on the look-ahead interpolation, Zhao et al. [54] proposed a real-time corner smoothing method, in which curvature continuous B-spline was adopted to replace the local corners. After using the bidirectional scanning algorithm for jerk limited S-shape feedrate profile, a real-time look-ahead interpolation was developed to obtain a feedrate profile with smoothing acceleration. The flow chart of look-ahead interpolator and motion controller can be found in Fig. 9. From this flow chart, the real-time look-ahead interpolations are usually divided into three modules, i.e.: geometric module, kinematics module and interpolation module, so that the real-time property can be satisfied through preprocessing the consecutive G01 blocks in the shared memory using these modules. Under these conditions, both the smoothness of tool path and feedrate can be guaranteed. Afterwards, Fan et al. [55], Sun et al. [56] and Huang et al. [57] also proposed the similar real-time corner smoothing method by locally replacing the corners of linear tool path spline curves. Xiao et al. [58] developed a different real-time corner smoothing method for five-axis short-segmented tool path using look-ahead interpolation. In their method, high allowable feedrate and high machining efficiency were achieved through merging the adjacent transition curves into an entire one under the given approximation error.

While for the works of real-time corner smoothing based on analytical representation, although balancing the high calculation efficiency and good kinematics performance is important for CNC machining, it is still difficult to develop the corner smoothing methods that can simultaneously guarantee the real-time and smoothing properties because of the highly nonlinear relation between the arc lengths and spline parameters. Theoretically, PH spline is able to ensure the above properties since the representation of arc length of PH curves is merely a polynomial function. It can facilitate an analytical reduction of the interpolation problem to the solution of a sequence of monotone polynomial equations, and a few Newton-Raphson iterations usually suffice to compute the reference points corresponding to the unique roots of these equations to high precision, allowing the real-time implementation [59]. Based on this, Hu et al. [60] developed a computationally efficient real-time corner smoothing algorithm. The linear tool path segments were smoothed by locally inserting the PH splines under the specified approximation error. Analytical solutions between the arc length and the curvature of the smoothed tool path were obtained through evaluating a polynomial function of the spline parameter, which is conducive to the real-time performance of the algorithm. Then, they further expanded their real-time corner smoothing method based on PH splines to the five-axis machining of linear tool path [61]. The tool tip position and tool orientation were both smoothed through employing C^3 continuous PH spline under the user-specified approximation error, and the algorithm also can be implemented in real-time environment due to its analytical representation.

In recent years, the real-time corner smoothing based on FIR filters has become more and more prevalent thanks to the development of FIR filtering technology. Different from the traditional geometry methods that the numerous linear G01 commands are smoothed by means of the arcs and splines, the kinematic corner smoothing algorithm using FIR filter can generate the real-time cornering trajectory of machine tool through filtering the discontinuous axis velocity commands at the junctions between adjacent linear tool path segments [62]. The contour errors at local corners can be controlled analytically by optimally overlapping acceleration profiles of the adjacent linear segments, and it can bring the significant improvement in the cycle time at the same time, even in generating the jerk-limited motion trajectories in real-time [63]. Tang et al. [64] proposed a real-time corner smoothing method with a FIR filtering algorithm to generate a smooth trajectory for CNC machine tools. Instead of inserting transition curves and solving feedrate scheduling between linear segments, nonstop feed motion was achieved in real-time by using the corner smoothing method based on FIR filtering. According to the kinematic equations and pulse commands of motion axes, the length of FIR filtering was determined to respect corner tolerance and kinematic constraints of the drives simultaneously. Rather than adding the dwell time in filter-based feedrate scheduling, two-axis motion was realized by evaluating corner speed and matching the same length of FIR filters, therefore, the cycle time and contouring performance can be improved. For further reducing the trajectory error caused by using smoothing filters for limiting acceleration and avoiding vibration of the machine tools, Ishizaki and Shamoto [65] proposed a real-time FIR-filtering based corner smoothing method, which simultaneously realized remarkable trajectory error reduction, vibration avoidance, and short machining time by smoothing the linear G01 commands according to the trajectory error and angular speed. Meanwhile, Tajima and Sencer [66] pointed out that corner smoothing using FIR low-pass filtering can blend very densely linear tool path within user-specified blending error and interpolate them in real-time under a minimal computational expense. The filtering-based real-time corner smoothing of five-axis machining tool paths can be found in Fig. 10. Due to the adoption of digital filtering (convolution), the corner smoothing method enables online interpolation of densely tool paths in a real-time and provides control over the frequency spectrum of interpolated trajectories at the same time.

4. Corner smoothing in machine tools and robots

After introducing the different types of corner smoothing methods, this section concentrates on the application differences be-

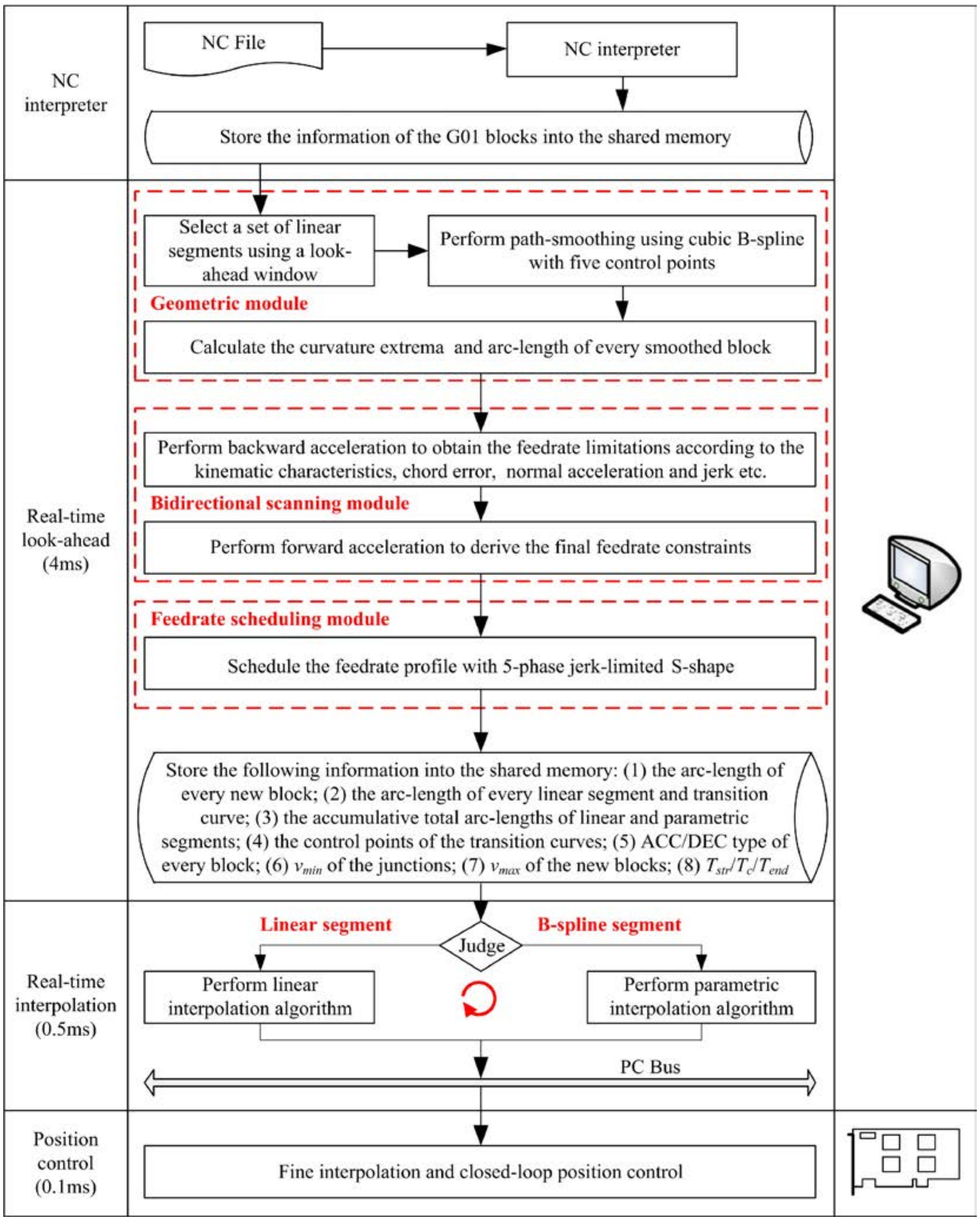


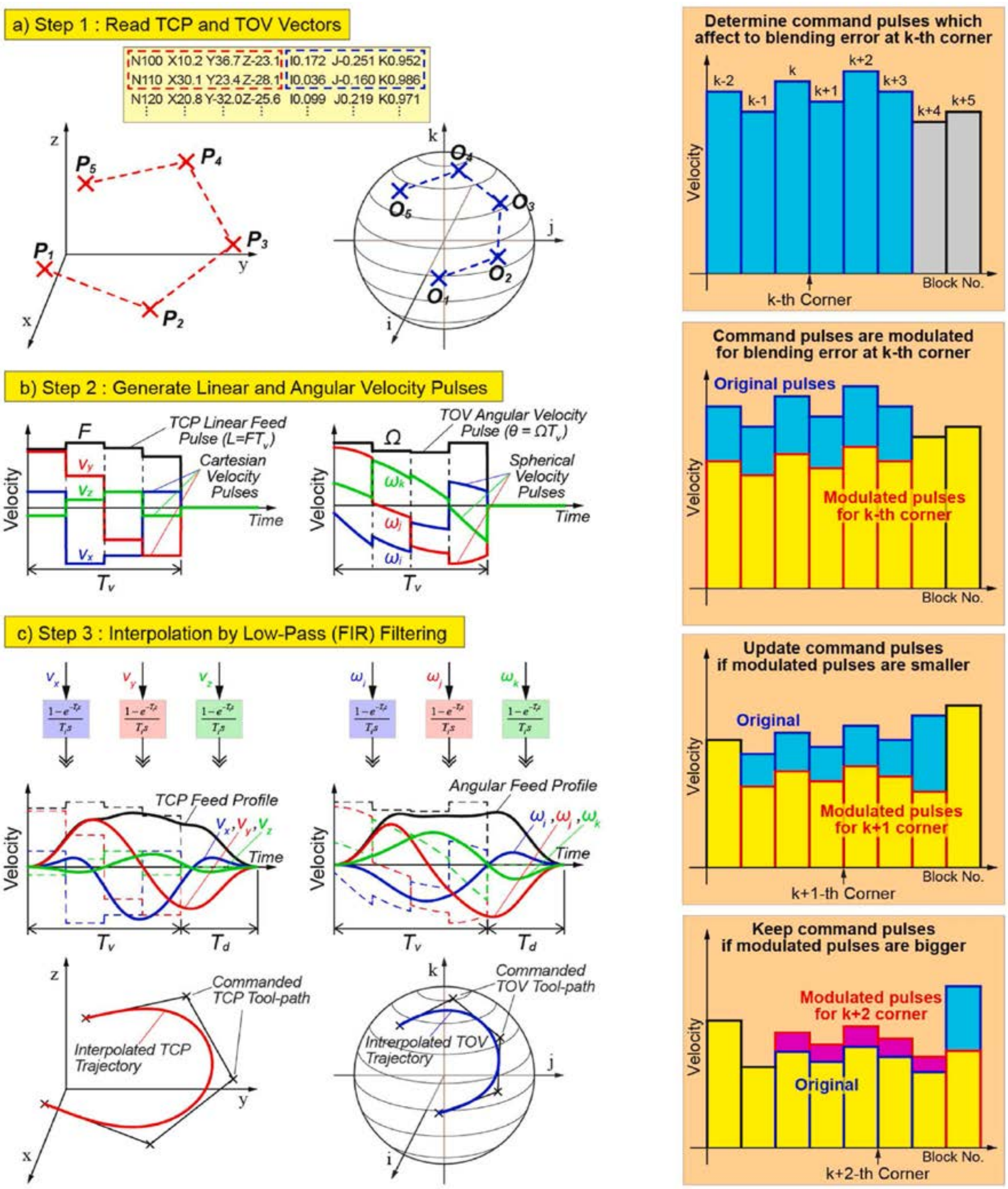
Fig. 9. Flow chart of the look-ahead interpolator and motion controller [54].

tween these methods in corner smoothing of machine tools and robots machining, and presents some successful application examples to show the recent advances and researches on corner smoothing.

4.1. Corner smoothing in CNC machining

At the early stage, corner smoothing is widely adopted in the pocket machining [92, 93], since lots of linear contour-parallel or Zig-Zag tool paths of pocket machining bring frequent acceleration and deceleration motions of machine tools, as illustrated in Fig. 11. The frequent acceleration and deceleration motions, even the full stop of machine tools at the corners, will directly affect the surface finish. Accordingly, these linear tool paths must be smoothed to improve the surface quality of parts, meanwhile, the trace left by machining has also become one of the criteria for judging the smoothness of tool path after corner smoothing. For verifying the corner smoothing indeed affects the surface quality, Wang et al. [12] provided the micro morphology of the parts machined by the tool path after corner smoothing and the linear tool path, respectively, as shown in Fig. 12. It can be intuitively seen that the tool path after corner smoothing has the better machining quality than that of the linear toolpath. Besides, as shown in Fig. 13, various tool paths were machined in the three-axis CNC machine tools, to illustrate the advantages of corner smoothing in improving surface quality. Obviously, it can be seen from Figs. 12 and 13, the corner smoothing plays an important role in affecting the surface finish. However for such three-axis corner smoothing, only the linear tool path in tool tip position needs to be paid attentions. A variety of methods or algorithms are proposed to achieve the corner-free and smooth tool path, the approximation error, chord error, and continuity requirement are the main factors to affect the smoothing results for the relatively long-segmented tool path, while for the short-segmented tool path, additional overlap problems have to be addressed. Besides of these geometry constraints, some researchers also pointed out that the smoothed linear tool path should also satisfy kinematics or dynamics conditions to obtain a better machining efficiency and quality.

However, for the corner smoothing of multi-axis machining, the smoothness of tool orientation and the synchronization between tool tip position and tool orientation need to be concerned in addition to maintaining the smoothness of tool tip position. Typically, to achieve the absolute smooth motions of machine tools, the tool tip position and tool orientation should be smoothed in the machine coordinate system, since if the linear tool path is smoothed in workpiece coordinate system, it still needs to be transformed into the machine coordinate system to carry out the real machining, such nonlinear transformation will make the originally smoothed tool path not smooth enough again. More importantly, the tool tip position and tool orientation are always smoothed in their own frame, the synchronization between tool tip position and tool orientation has to be considered to avoid the abrupt changes of tool orientation at the junctions of inserted transitions and remaining linear segments, which has become a common problem in corner smoothing of multi-axis machining. For different corner smoothing methods, the existing synchronization strategies are also different. For example,



(a) (b)

Fig. 10. Filtering-based real-time corner smoothing of five-axis machining tool paths [66]. (a) Filtering-based linear interpolation of 5-axis machining tool path; (b) Windowing approach for real-time blending error control.

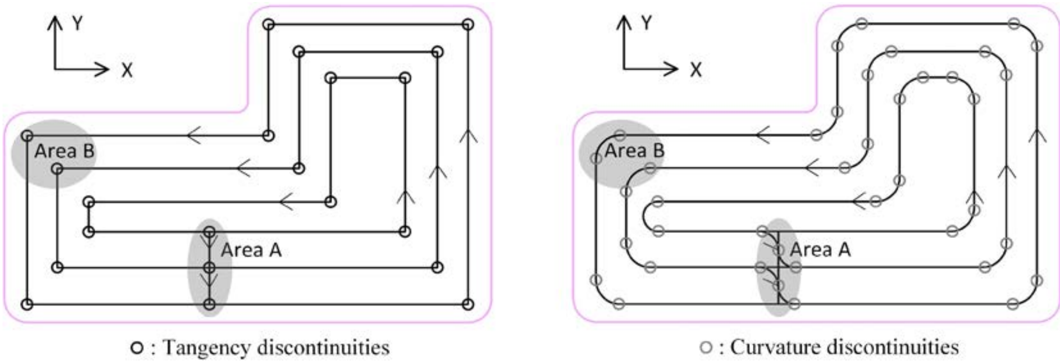


Fig. 11. Corner smoothing in pocket machining [92].

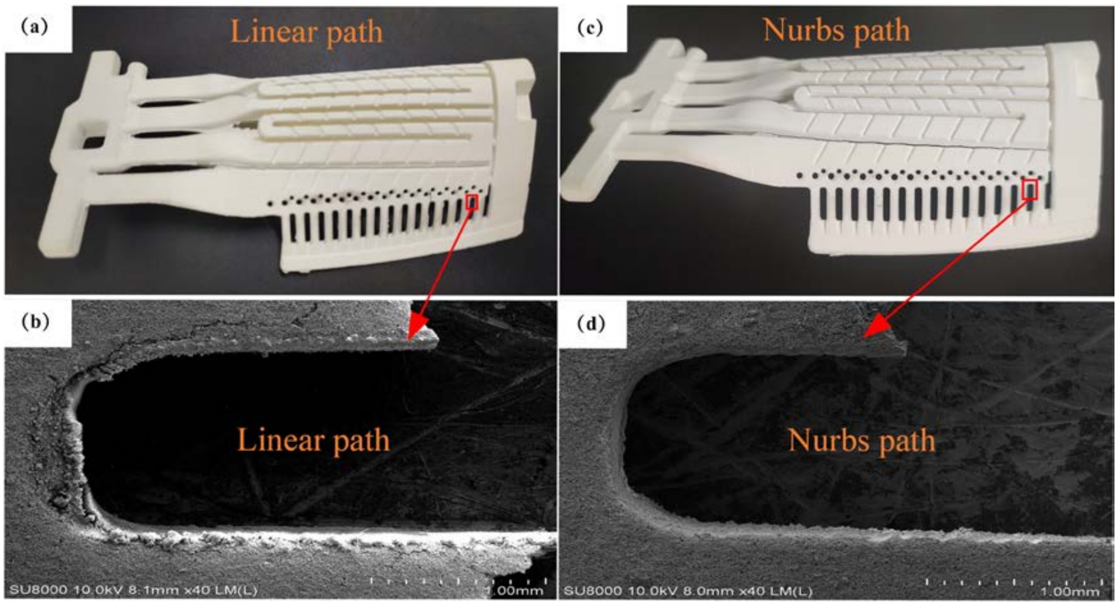


Fig. 12. Effects of different tool paths on surface quality [12]. (a) Linear tool path; (b) Surface quality of part machined by linear tool path; (c) NURBS tool path after corner smoothing; (d) Surface quality of part machined by smoothed NURBS tool path.

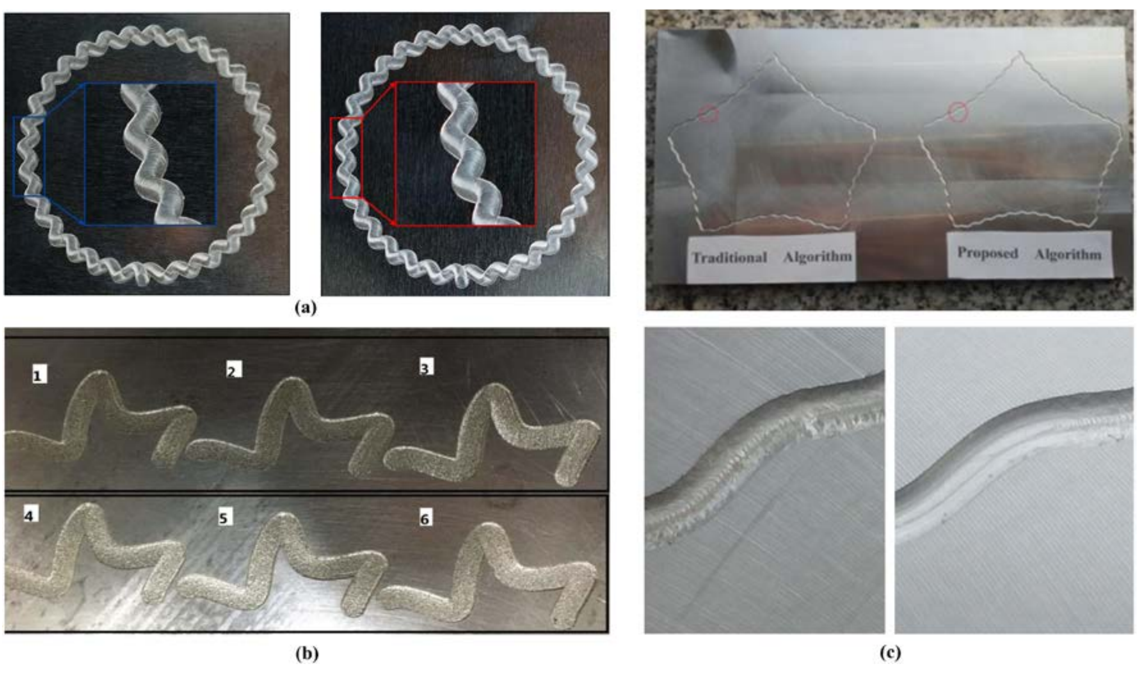


Fig. 13. Surface finish comparisons in three-axis machining, which the smoothed one can have better surface finish. (a) Surface finish after real machining using different corner smoothing methods reported in Ref. 11; (b) Surface finish after real machining using different corner smoothing methods reported in Ref. 5; (c) Surface finish after real machining using different corner smoothing methods reported in Ref. 56.

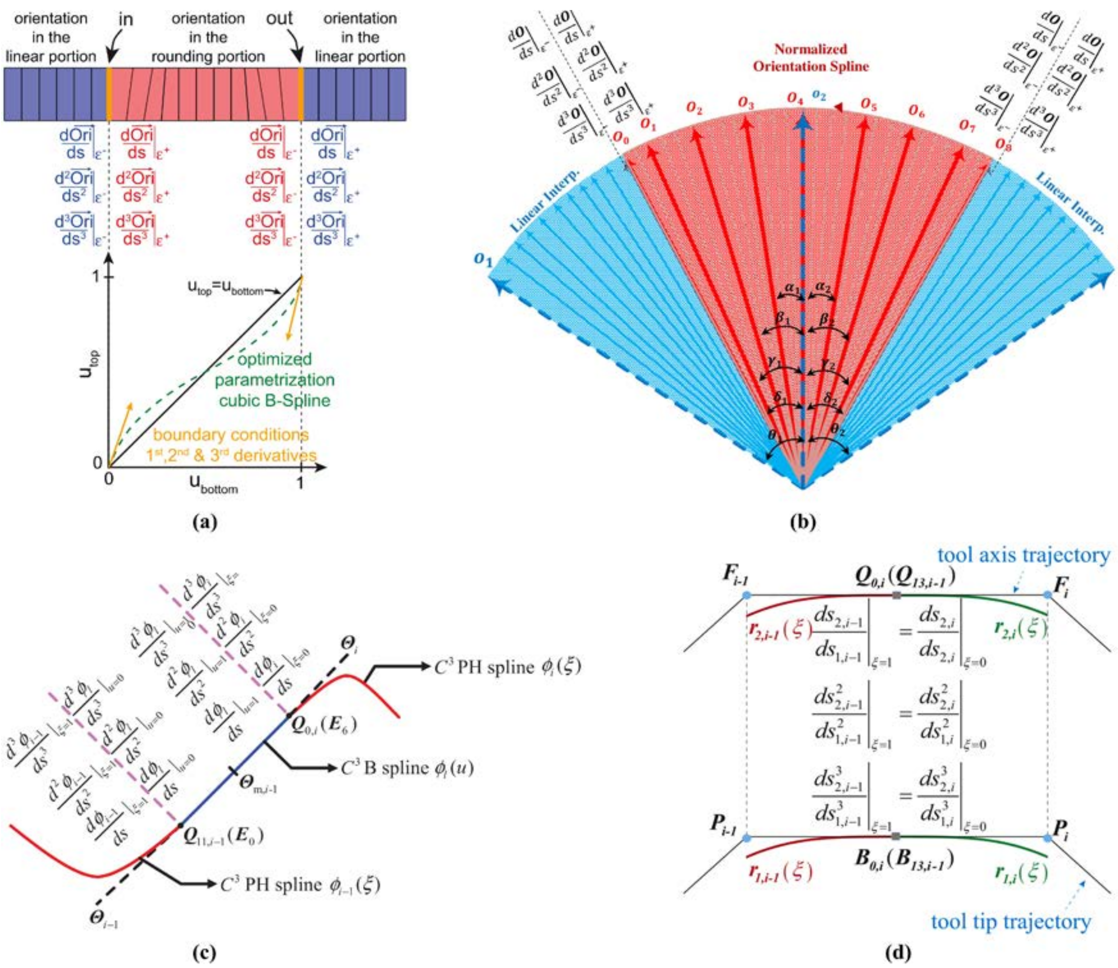


Fig. 14. Different parameter synchronization strategies in corner smoothing of five-axis linear tool path. (a) Parameter synchronization at the connection using an additional correction cubic B-spline [75]; (b) Parameter synchronization considering the continuity between linear interpolation and normalized orientation spline [13]; (c) Parameter synchronization by transforming the remaining linear segment into a linear spline while guaranteeing the continuous changes of tool orientation w.r.t. the tool tip position displacement [61]; (d) Parameter synchronization is aimed at guaranteeing the continuous tool orientation varying rate with respect to the tool tip displacement [43].

corner smoothing based on geometry methods usually adopt the additional correction spline and parameter synchronization, as shown in Fig. 14. The additional correction spline is used to establish the connection between the parameter of transition curves in tool tip position and tool orientation, while the parameter synchronization is achieved by guaranteeing the continuous changes of tool orientation w.r.t. the tool tip position displacement. When synchronizing the tool tip position and tool orientation, the kinematics performance of machine tools can be improved significantly, which has been proved by existing works in Refs. 43, 51, 61. In kinematics corner smoothing method, instead of generating additional correction spline or parameter synchronization, the motions between tool tip position and tool orientation along corner trajectory are synchronized through creating the proportional relationship between their velocities. After determining the velocities of five motion axes along corner trajectory, the tool tip and tool orientation can be generated smoothly and synchronized [90]. Moreover, the synchronization in FIR filter based corner smoothing also needs to be considered. Usually, the linear translational motions of the tool tip position are interpolated by FIR filter of Cartesian velocity pulses in G01 segments, and to generate the constant rotation speed of tool orientation, spherical linear interpolation is used to directly filter the unit tool orientation vectors in the spherical coordinates. Precise tool motion synchronization is realized by matching time-constants of FIR filters utilized for translational and rotational interpolation [66]. After smoothing the tool path in tool tip position and tool orientation with the synchronization, the geometrically corner-free and kinematically smooth tool path can be generated, thereby leading to a better surface finish, as shown in Fig. 15. In addition, the tool path after corner smoothing also can be employed in machining the complex curved surface parts, such as impeller in Fig. 16, to improve the kinematics performance of machine tool during the entire machining processes.

4.2. Corner smoothing in robotic machining

Nowadays, robots have been widely employed in industrial machining thanks to their low cost, large operation reaching space and high machining flexibility. Similar to the CNC machining, the smooth tool path is also important for pursuing the superior kinematics and dynamic performances of robot manipulators. However, the corner smoothing methods in five-axis machining need to be modified to adapt to the robot machining because the mechanical configurations and kinematics structures between machine tools and robots are different, as shown in Fig. 17. Similar to the corner smoothing in five-axis machining, the corner smoothing in robot smoothing should also take into account the smoothness of tool tip

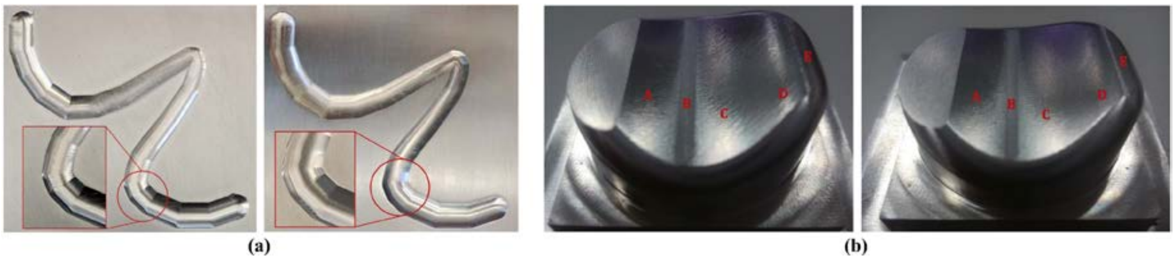


Fig. 15. Surface finish after corner smoothing of five-axis linear tool path. (a) Fan-shaped tool path after real machining using different corner smoothing methods, which the left figure uses the traditional G01 tool path and the right figure uses the corner smoothing method in Ref. 51; (b) Parts after real machining using different corner smoothing methods, which the left figure uses the traditional corner smoothing method and the right figure uses the corner smoothing method proposed in Ref. 55.

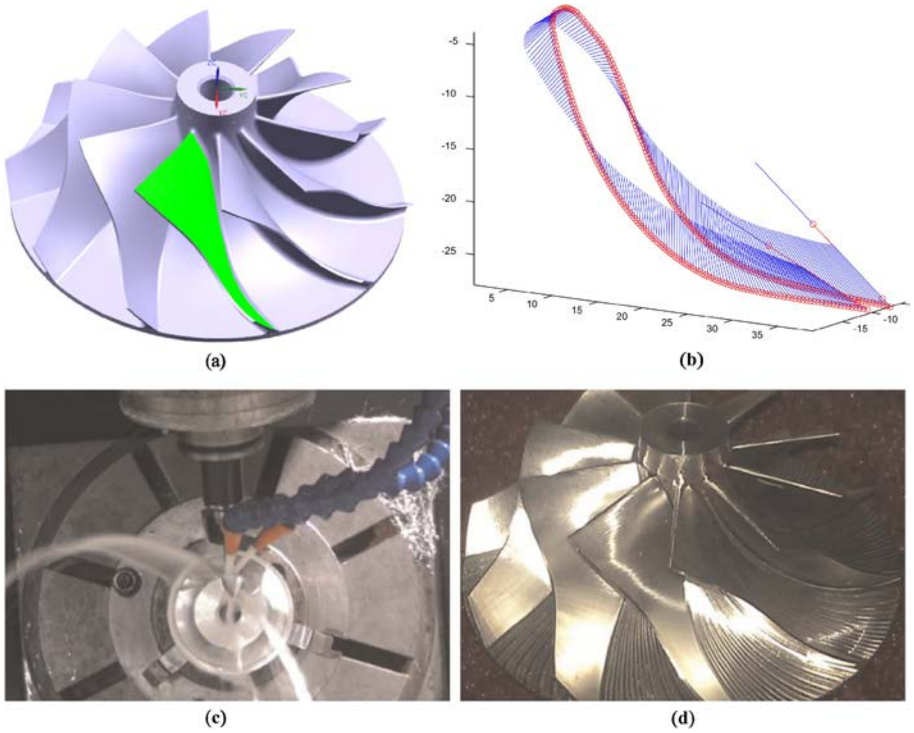


Fig. 16. Corner smoothing used in five-axis machining of an impeller [42]. (a) Impeller model used in real machining; (b) Tool path after smoothing the corners in tool tip position and tool orientation; (c) Machining process of the impeller; (d) Machined impeller after using corner smoothing.

position and tool orientation, as well as their parameter synchronization. While the difference is that the transformation relationship between robot pose and joint motion is much more complex than that of the machine tools, thereby, the motion smoothness of robot joints is a specially considerable factor in corner smoothing of robot machining. In Ref. 81, Yang et al. proposed an analytical C^3 continuous tool path corner smoothing algorithm for robot manipulators with six rotational (6R) joints. The tool tip position was smoothed directly in the workpiece coordinate system, while the tool orientation was smoothed after transferring the tool orientation matrix as three rotary angles. In order to guarantee the smooth changes of tool orientation w.r.t. the tool tip position displacement, the tool tip position and tool orientation were also synchronized. In addition, to achieve a smooth motion of robot machining, Peng et al. [82] presented an analytical decoupled C^3 continuous local path smoothing method for industrial robots, where the tool top position was smoothed in the reference coordinate system while the tool orientation was smoothed in the rotation parametric space based on the exponential coordinates of rotations. Moreover, to improve the smoothness of the angular motion on the remaining linear segments, the parameter synchronization of the tool tip position and tool orientation was guaranteed by sharing the same spline parameter. Furthermore, to obtain a smoother motions of robot joints, Liu et al. [85] proposed a C^3 continuous corner smoothing method for the hybrid machining robot, where the tool tip position and tool orientation were directly smoothed through inserting the quantic B-splines, and their parameter synchronization was also guaranteed by converting the remaining linear segment into a specially designed 7th-degree B-spline, As shown in their real machining testing in Fig. 18, the smoothed tool path can achieve the better surface finish. Different from above geometry based corner smoothing methods, which often need two steps to schedule the motions of robots manipulator, Sun et al. [84] proposed a corner smoothing method based on the FIR filters to generate the smooth tool tip position and tool orientation trajectory in one step for 6R robot manipulators. In their method [81], the maximum velocity and acceleration of joints were constrained when calculating the time parameters of the FIR filter, while the tool tip position and tool orientation deviations were all constrained by specially designing the overlapping time at corners, thereby, guaranteeing the kinematic constraints of the joints and maintaining a better tracking performance in motion control process.

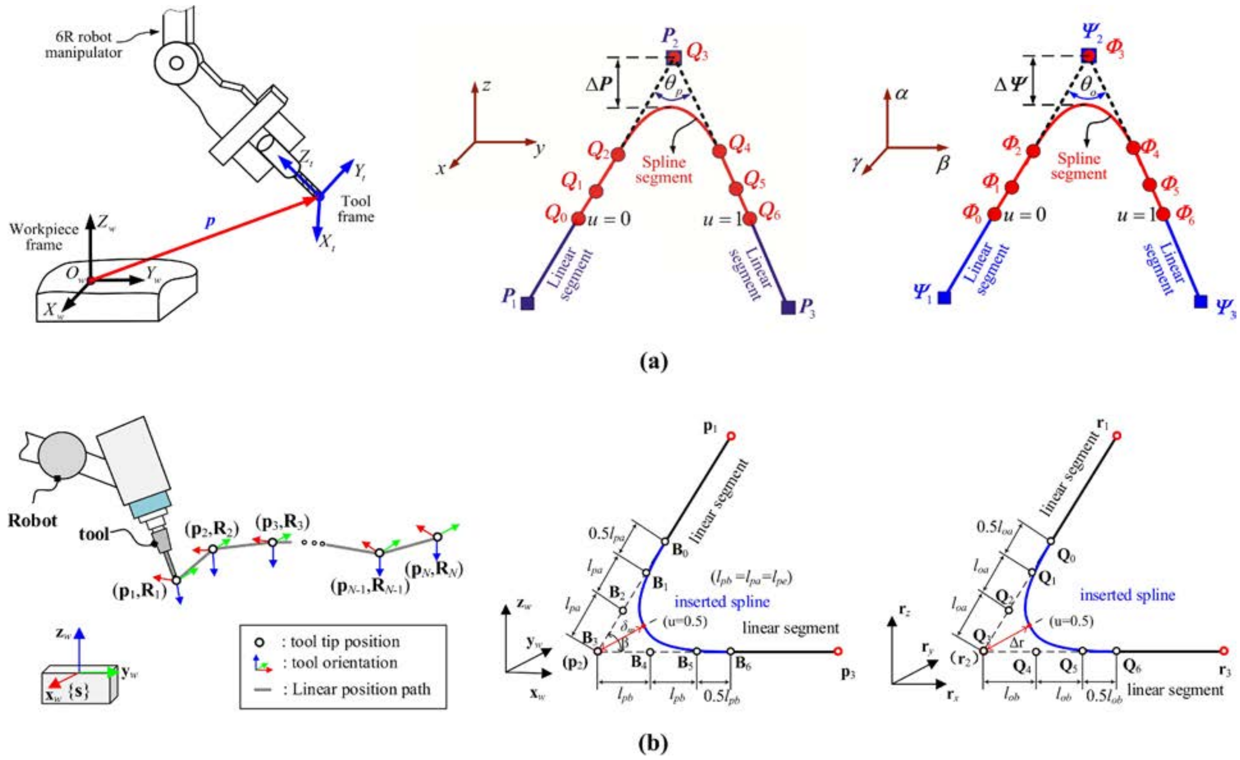


Fig. 17. C^3 **continuous tool path corner smoothing of linear tool path in robot machining. (a) The tool tip position smoothed in the workpiece coordinate system and the tool orientation is smoothed after transferring the tool orientation matrix as three rotary angles [81]; (b) The tool position path is smoothed in the reference system while the tool orientation is smoothed in the rotation parametric space based on the exponential coordinates of rotations [82].**

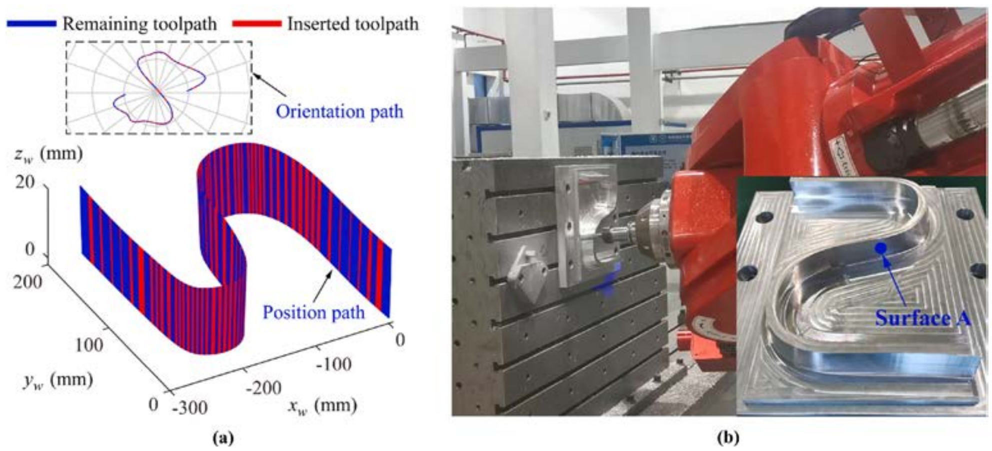


Fig. 18. Real machining of a s-shaped tool path using corner smoothing in robotic machining [85]. (a) The test s-shaped tool path after corner smoothing using the method in Ref. 85; (b) Real machining testing of the s-shaped tool path and its machined surface.

5. Conclusions

In this paper, the existing corner smoothing methods are briefly reviewed from the perspective of different categories. It has been noticed that numerous methods and techniques have been studied in depth, resulting in significant development and improvement in tool path smoothing, however, some challenging issues still need to be further studied so as to further improve the kinematics and the dynamics performances. Based on the analysis of the existing research progress in previous sections, the conclusions, remaining challenges and future directions in corner smoothing of the linear tool path could be:

(1) The existing corner smoothing methods can achieve well the smoothness of linear tool path through various strategies, and are no longer limited to the geometrically corner-free tool path. The developed algorithms that have analytical representation without any iterative, real-time property in interactivity, and the transitions with improved kinematics performance of machine tools are more preferred.

(2) Although the existing corner smoothing methods can schedule well the motions of the machine tool, few of them explored the influence of kinematics performance difference of each drive axis on corner smoothing, thereby resulting in a near optimal smoothing results. Meanwhile, the kinematically collaborative scheduling between tool tip position and tool orientation are not fully considered.

(3) Corner smoothing methods based on FIR filter are more

promising in smoothing the linear tool path segments compared with the traditional methods. Besides, corner smoothing in robot machining still needs further exploration in the foreseeable future since the motions of robot joints are more complicated than machine tools. Using the direct motion planning and the FIR filter may be the effective strategies to obtain the continuous motions of robots.

(4) Finally, it still remains a challenge to develop an embedded corner smoothing function that can cooperate with the existing CNC systems with superior performance.

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